Entanglement entropy in higher spin gravity

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based on work in collaboration with

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Gauge Gravity Duality 2013

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Outline

Why Higher Spin gravity in context of AdS/CFT?

- From conceptional point of view:

  What is the gravity dual of non-interaction field theoies? Of minimal CFTs?
- For condensed matter applications:

  Higher Spin Gavity in 4D dual to O(N) models in the large N-limit.

  How do we compute entanglement entropy in higher spin gravity?
- For (quantum) gravity applications:

  Higher Spin Gravity as toy-model to study propeties of black holes in asymptotically AdS.

  Can we study black hole creation and evaporation explicitly since we have both sides under full control?

  What is geometry in higher spin gravity?

We will see that both questions are connected. In particular I hope to convince you that we may learn something about geometry, causal structure, event horizons, etc. by studying entanglement entropy.

Why Higher Spin gravity in context of AdS/CFT?

In this talk:

I focus on higher spin gravity in three spacetime dimensions.

advantage:

We do not have to take into account the infinite tower of higher spins since we can truncate to a finite order.

Here:

I consider only 'minimal' extensions of Einstein Gravity by adding a spin-3 degree of freedom.

Review: 3D Gravity as Chern-Simons theory

Action

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}(R + \frac{2}{l^2}) - \int_{\partial\mathcal{M}} \omega^a \wedge e_a$$

or equivalently

$$S = S_{CS}[A] - S_{CS}[\overline{A}] \qquad A = \omega + e/l\,, \quad \overline{A} = \omega - e/l$$

$$S_{CS}[A] = \frac{k}{4\pi}\int \mathrm{Tr}\left[A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right]$$

with gauge fields $A, \overline{A} \in sl(2,\mathbb{R})$ and Chern-Simons level, $k = \frac{l}{4G}$, l the radius of curvature of AdS, which we set to one, $l = 1$.

Equations of motion

$$F = dA + A \wedge A = 0\,, \qquad \overline{F} = d\overline{A} + \overline{A} \wedge \overline{A} = 0$$

3D Higher Spin Gravity as Chern-Simons theory

We only want to add a spin-3 field, so what do we have to modify

3D Gravity coupled to spin-3 field given by

$$S = S_{CS}[A] - S_{CS}[\overline{A}] \qquad A = \omega + e/l\,, \quad \overline{A} = \omega - e/l$$

$$S_{CS}[A] = \frac{k}{4\pi} \int \mathrm{Tr} \left[A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right]$$

with gauge fields $A, \overline{A} \in sl(3, \mathbb{R})$

3D Higher spin gravity as Chern-Simons theory with gauge group $SL(3, \mathbb{R}) \times SL(3, \mathbb{R})$

Equations of motion

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3D Higher Spin Gravity as Chern-Simons theory II

Theory has two different AdS vacua depending of an $\mathfrak{sl}(2,\mathbb{R})$ embedding into $\mathfrak{sl}(3,\mathbb{R})$.

$\mathfrak{sl}(3,\mathbb{R})$ generators

$\mathfrak{sl}(3,\mathbb{R})$ has eight generators which we split into:

- L_{-1}, L_0, L_1 generators with commutation relations $[L_i, L_j] = (i-j)L_{i+j}$
- $W_j,\ (j = -2, -1, ..., 2)$ satisfying $[L_j, W_m] = (2j - m)W_{j+m}$

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inequivalent embeddings of $\mathfrak{sl}(2,\mathbb{R})$ into $\mathfrak{sl}(3,\mathbb{R})$

principle embedding:

take $J_a = L_a$ as $\mathfrak{sl}(2,\mathbb{R})$ generators

bulk degrees of freedom: metric $g_{\mu\nu}$ and Spin-3 field $\phi_{\mu\nu\rho}$ given by

$$g_{\mu\nu} = \frac{1}{2}\mathrm{tr}_f(e_\mu e_\nu)\,, \qquad \phi_{\mu\nu\rho} = \frac{1}{6}\mathrm{tr}_f(e_{(\mu}e_\nu e_{\rho)}) \qquad e = e_\mu dx^\mu .$$

Asymptotic symmetry algebra: $\mathcal{W}_3 \times \mathcal{W}_3$ [Campeleoni, Fredenhagen, Penninger, Theisen, '10]

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inequivalent embeddings of $\mathfrak{sl}(2,\mathbb{R})$ into $\mathfrak{sl}(3,\mathbb{R})$

diagonal embedding:

take $J_0 = L_0/2,\ J_{\pm 1} = \pm W_{\pm 2}/4$ as $\mathfrak{sl}(2,\mathbb{R})$ generators

bulk degrees of freedom: spin-2 field, a pair of spin-1 U(1) gauge fields and of spin 3/2 bosonic fields

Asymptotic symmetry algebra: $\mathcal{W}_3^{(2)} \times \mathcal{W}_3^{(2)}$

3D Higher Spin Gravity as Chern-Simons theory II

Gauge connection for AdS in Poincare patch

$$A = A_+\, dx^+ + A_-\, dx^- + J_0\, d\rho, \qquad \overline{A} = \overline{A}_+\, dx^+ + \overline{A}_-\, dx^- - J_0\, d\rho$$

$$A_+ = e^\rho J_1, \qquad \overline{A}_- = -e^\rho J_{-1}, \qquad A_- = \overline{A}_+ = 0$$

where $x^\pm = t \pm \phi$ and ρ is the radial direction

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Gauge Transformation

$$A \to g^{-1} A\, g + g^{-1}\, dg \qquad \overline{A} \to \tilde{g}\, \overline{A}\, \tilde{g}^{-1} - d\tilde{g}\, \tilde{g}^{-1}$$

where g and $\tilde{g}$ are functions of spacetime coordinates and are valued in $SL(3, \mathbb{R})$.

3D Higher Spin Gravity as Chern-Simons theory II

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Remarks

- Some of the gauge transformations (namely $g, \tilde{g} \in SL(2, \mathbb{R}) \subset SL(3, \mathbb{R})$) correspond to diffeomorphisms.
- Higher spin gauge transformations may change the causal structure of the spacetime. What is the notion of geometry in higher spin gravity?

Black holes in 3D Higher Spin Gravity I

Can we find black holes in 3D Higher spin gravity? *Yes, ...*

- BTZ black hole is also a solution of 3D higher spin gravity.

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- There exist also black holes with higher spin charge [Gutperle, Kraus, '11, MA, Gutperle, Kraus, Perlmutter, '11]

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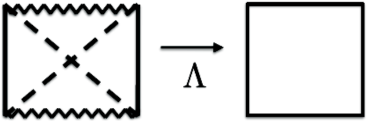
The causal structure is not invariant under higher spin transformations. [MA, Gutperle, Kraus, Perlmutter, '11]



For example, a higher spin black hole in one gauge can look like a traversable wormhole in another gauge, even though they describe the same physics.

Black holes in 3D Higher Spin Gravity II

Thermodynamics of charged higher spin black holes are only consistent if Holonomy condition is satisfied.

The Holonomy condition

The holonomies associated with the Euclidean time circle

$$\omega = 2\pi(\tau A_+ - \overline{\tau} A_-) \qquad \overline{\omega} = 2\pi(\tau \overline{A}_+ - \overline{\tau} \overline{A}_-)$$

have eigenvalues $(0, 2\pi i, -2\pi i)$ as in the case of the BTZ black hole.

Gauge invariant characterization of higher spin black holes!

There is another interesting black hole in diagonal embedding [Castro, Hijano, Lepage-Jutier, Maloney, '11]

Review: Entanglement entropy in CFT & AdS/CFT

Entanglement entropy in CFT

Consider quantum system described by a density matrix ϱ, and divide it into two subsystems A and $B = A^c$. Reduced density matrix ϱ_A of subsystem A:

$$\varrho_A = \mathrm{Tr}_{A^c}\, \varrho$$

Entanglement entropy S_{EE} = von Neumann entropy associated with ϱ_A:

$$S_{EE} = -\mathrm{Tr}_A \varrho_A \log \varrho_A \,.$$

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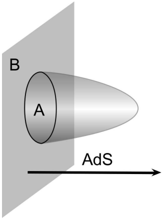
Gravity dual of entanglement entropy (supergravity limit)



Construct minimal spacelike surface $m(A)$ which is anchored at the boundary ∂A of the region A and extends into the bulk spacetime.

$$S_{EE} = \frac{m(A)}{4G_N} \,.$$

Entanglement entropy in higher spin gravity I

Geodesics will not work: What is spacetime geometry in higher spin gravity? Can we find a bulk object that correctly calculates the entanglement entropy?

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Proposal for Entanglement Entropy in Higher Spin Gravity [MA, Castro, Iqbal, '13; see also de Boer, Jottar,'13 for a similar proposal]

Entanglement Entropy may be calculated from a Wilson line in infinite dim. rep.

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}(\mathcal{P} \exp \int_{\mathcal{C}} \mathcal{A}) = \int \mathcal{D}U \exp(-S(U, P; \mathcal{A})_C)$$

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- $\mathcal{R}$ contains information about quantum numbers of probe
- $U(s) \in SL(3, \mathbb{R})$: field capturing the dynamics of the probe
- $P(s) \in \mathfrak{sl}(3, \mathbb{R})$: momentum conjugate to $U(x)$

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where $D_sU = \frac{d}{ds}U + A_sU - U\overline{A}_s$, $\quad A_s \equiv A_\mu \frac{dx^\mu}{ds}$,

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Entanglement entropy: Take $c_3 = 0$ and $\sqrt{2c_2} \to \frac{c}{6}$ and compute $S_{EE} = -\log(W_{\mathcal{R}}(\mathcal{C}))$

Entanglement entropy in higher spin gravity II

Infinite dimensional highest-weight state $|h, w\rangle$ with definite eigenvalues under the elements of the $SL(3, \mathbb{R})$ Cartan L_0, W_0:

$$L_0|h, w\rangle = h|h, w\rangle \ , \qquad W_0|h, w\rangle = w|h, w\rangle \ ,$$

and which is annihilated by the positive modes of the algebra:

$$L_1|h, w\rangle = 0 \ , \qquad W_{1,2}|h, w\rangle = 0 \ .$$

We may now generate other excited states by acting with L_{-1}, $W_{-1,-2}$ on this ground state, filling out an infinite dimensional unitary and irreducible representation.

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Relationship between Casimirs c_2, c_3 and h, w

$$C_2 = \tfrac{1}{2}L_0^2 + \tfrac{3}{8}W_0^2 + \cdots \ , \qquad C_3 = \tfrac{3}{8}W_0\left(L_0^2 - \tfrac{1}{4}W_0^2\right) + \cdots \ .$$

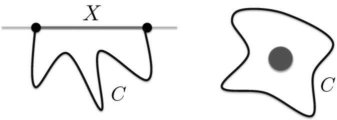
Acting with C_2 and C_3 on the highest weight state $|h, w\rangle$ we find

$$c_2 = \tfrac{1}{2}h^2 + \tfrac{3}{8}w^2 \ , \qquad c_3 = \tfrac{3}{8}w\left(h^2 - \tfrac{1}{4}w^2\right) \ .$$

We consider heighest weight representation with $w = 0, h = c/6$ implying $c_3 = 0$.

Entanglement entropy in higher spin gravity III

Two possible choices for C:



Open Wilson Line determines the entanglement entropy of interval X

Wilson Loop computes the thermal entropy of a black hole

Entanglement entropy in higher spin gravity III

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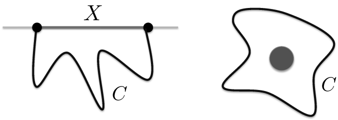
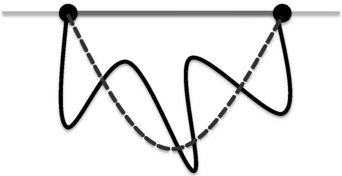


Open Wilson Line determines the entanglement entropy of interval X

Wilson Loop computes the thermal entropy of a black hole

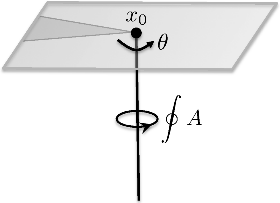
Wilson Line does not depend on path. Geodesic equation is irrelevant to reproduce proper distance.



Overview: Checks for our proposal

Why do we think our proposal is correct?

- In the case of CS theory with $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$: Entanglement entropy determined by geodesics.
- Perfect agreement with CFT results (where available)
- Wilson line induces conical defect if backreaction is included [see also Lewkowycz, Maldacena,'13]



Result of this argument: $\sqrt{2c_2} \to \frac{c}{6}$

Checks for our proposal I

How to get the geodesics for 3D spin-2 gravity descibed by CS theory with $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$?

$$S(U,P;\mathcal{A})_C = \int ds \left(\mathrm{Tr}\left(PU^{-1}D_sU\right) + \lambda(s)\left(\mathrm{Tr}(P^2) - c_2\right)\right) ,$$

The equations of motion reduce to

$$U^{-1}D_sU + 2\lambda P = 0 , \quad \frac{d}{ds}P + [\bar{A}_s, P] = 0 . \quad \mathrm{Tr}(P^2) = c_2$$

Plugging the eom back into the action we obtain

$$S(U;\mathcal{A})_C = \sqrt{c_2}\int_C ds\sqrt{\mathrm{Tr}\,(U^{-1}D_sU)^2}$$

The eom with respect to $U(s)$ are

$$\frac{d}{ds}\left((A^u - \bar{A})_\mu \frac{dx^\mu}{ds}\right) + [\bar{A}_\mu, A^u_\nu]\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \qquad A^u_s \equiv U^{-1}\frac{d}{ds}U + U^{-1}A_sU$$

provided s is the proper distance (reparameterization invariance!)

Checks for our proposal II

For spin-two gravity, $U(s) = \mathbb{1}$ is a solution
(but not for higher spin gravity in generic gauge field backgrounds)

Geodesic equation

$$\frac{d}{ds}\left((A-\overline{A})_\mu \frac{dx^\mu}{ds}\right) + [\overline{A}_\mu, A_\nu]\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0$$

Proper distance appears in on-shell action

$$\begin{aligned} S_C &= \sqrt{c_2}\int_C ds \sqrt{\mathrm{Tr}\left((A-\overline{A})_\mu (A-\overline{A})_\nu \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\right)} \\ &= \sqrt{2c_2}\int_C ds \sqrt{g_{\mu\nu}(x)\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}} \equiv \sqrt{2c_2} L_C \end{aligned}$$

and thus

$$W_{\mathcal{R}}(C) = e^{-\sqrt{2c_2}L_C}$$

or, for the entanglement entropy we get

$$S_{EE} = -\log W_{\mathcal{R}}(C) = \sqrt{2c_2} L_C = \frac{1}{4G} L_C$$

using $\sqrt{2c_2} = \frac{c}{6} = k = \frac{1}{4G}$

Checks for our proposal III

Result for the open interval at zero temperature (i.e. in AdS)

$$S_{EE} = \frac{c}{3} \log\left(\frac{\Delta\phi}{\epsilon}\right),$$

where $\epsilon \equiv e^{-\rho_0}$ and $\Delta\phi = \phi(s = s_f) - \phi(s = 0)$

Results for the open interval at finite temperature (i.e. in BTZ)

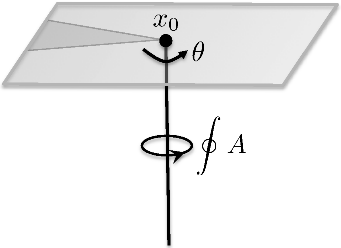
$$S_{EE} = \frac{c}{3} \log\left(\frac{\beta}{\pi\epsilon} \sinh\left(\frac{\pi\Delta\phi}{\beta}\right)\right) .$$

Results for the thermal entropy (BTZ black hole)

$$S_{th} = -\log W_R(C) = 2\pi\sqrt{2\pi k\mathcal{L}} + 2\pi\sqrt{2\pi k\bar{\mathcal{L}}} .$$

Checks for our proposal IV

Consider interval $A = [x_0, x_1]$ and determine backreaction of Wilson line near x_0 [see also Lewkowycz, Maldacena,'13]



metric near x_0

$$ds^2 = d\rho^2 + e^{2\rho}\left(dr^2 + r^2\left(\frac{\sqrt{2c_2}}{k} - 1\right)^2 d\theta^2\right)$$

Checks for our proposal IV

Deficit angle has to be $2\pi/n$ and thus for $n \to 1$

$$\sqrt{2c_2} = k(n-1) = \frac{c}{6}(n-1)\,.$$

Determining the entanglement entropy from Renyi entropies,

$$S_{EE} = \lim_{n\to 1} \frac{1}{1-n} \log \mathrm{Tr}\rho_A^n = \lim_{n\to 1} \frac{1}{1-n} \log(W_{\mathcal{R}_n}(C))$$

we get finally

$$S_{EE} = -\lim_{n\to 1} \frac{1}{1-n} \sqrt{2c_2} L_C = \frac{c}{6} L_C\,,$$

where we assumed

$$W_{\mathcal{R}_n}(C) = e^{-\sqrt{2c_2}L_C}$$

Summary

In this talk we focused on gravity + spin-3 field in AdS_3 and put forward the proposal:

Entanglement entropy in CFT = Wilson Line in CS theory in inf. dim representation

- Proposal passes non-trivial checks:
  collapses to geodesics for spin-2 gravity, reproduces CFT results
  Wilson line creates correct conical deficit
- Results which I have not shown:
  Thermal Entropy for higher spin black holes [agrees with De Boer, Jottar,'13]

possible Generalizations

- Generalizations to TMG, to $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$ and to Vasiliev theory
- Renyi entropies, more than one interval, ...

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For more details

Wilson Lines & Entanglement Entropy in higher spin gravity
MA, Castro, Iqbal, arXiv: 1306.4338

or just ask me!

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- Renyi entropies, more than one interval, ...

For more details

Wilson Lines & Entanglement Entropy in higher spin gravity
MA, Castro, Iqbal, arXiv: 1306.4338

or just ask me!

Thank you!