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A universal fermionic analogue of the shear viscosity

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We show that a hydrodynamic transport coefficient, naturally defined in the low frequency, long wavelength limit of any supersymmetric field theory at finite temperature, possesses very similar universal properties as the shear viscosity, when the theory has a dual description in terms of classical AdS black brane (super-)gravity. Via a Kubo formula it can be computed from the low frequency, low momentum pole of the retarded Green's function of supersymmetry currents. This pole describes the phonino mode, the massless Goldstone fermion of spontaneous supersymmetry breaking by temperature. The correlator can then be related to a universal fermionic absorption cross section result derived by Das, Gibbons and Mathur '96.
Our description allows for the computation of supercharge diffusion constants in all dimensions which we independently check using the methods of Policastro '08 and Policastro and Kontoudi '12.

SUPERSYMMETRY

Connection to universal absorption cross section:

$$\epsilon D_{3/2} = \frac{1}{4\pi G}\sigma_{1/2} \quad \leftrightarrow \quad \eta = \frac{1}{16\pi G}\sigma_0 \;\Rightarrow\; \frac{\eta}{s} = \frac{1}{4\pi}$$

Explicitly computed for non-dilatonic AdS_{d+1} black branes.

HYDRODYNAMICS

Motivation

Holographic hydrodynamics has been very successful at calculating **transport coefficients** for strongly coupled systems, in particular with regard to applications to real-world systems such as the quark-gluon plasma. One of the most far-reaching results obtained in this context is the universality of the ratio of shear viscosity and entropy density.
In a further recent line of investigation within gauge / gravity duality, new results for **fermionic correlators** have been found in models which describe strongly coupled systems that are interesting in view of applications to condensed matter physics.
This naturally leads to the question if from these correlators we may find similar universal results.
In supersymmetric field theories the (fermionic) **supersymmetry** current lies in a multiplet together with the energy momentum tensor, from which one may compute the shear viscosity. So, can one define a hydrodynamic transport coefficient from the supersymmetry current which has similarly universal properties as the shear viscosity?

Supersymmetric Hydrodynamics and Kubo formula

The IR of a supersymmetric theory with SUSY breaking by temperature ("supersymmetric hydrodynamics") may be described as the effective theory of the phonino, the Goldstone fermion of spontaneous SUSY breaking by temperature, and the normal fluid (Hoyos, Keren-Zur, Oz '12). Although there are no classical fermionic charges (!), one may obtain a gradient expansion of the supersymmetry current in terms of the supercharge density $\rho = S^0$:

$$S^i_{\text{diss}} = -D_{3/2}\left(\delta^i_j - \frac{1}{d-1}\gamma^i\gamma_j\right)\nabla^j\rho - D_{1/2}\gamma^i(\gamma\cdot\nabla)\rho$$

The transport coefficient $D_{3/2}$, which is related to the diffusion constant in the sound-like dispersion relation of the phonino, appears in front of the transverse, γ-traceless ($\leftrightarrow$ helicity 3/2) part of the vector-spinor $\nabla^j\rho$, the way the shear viscosity appears in front of the symmetric traceless ($\leftrightarrow$ helicity 2) part of the transverse energy momentum tensor:

$$T^{ij}_{\text{diss}} = -\eta\left(\delta^{ik}\delta^{jl} + \delta^{jk}\delta^{il} - \frac{2}{d-1}\delta^{ij}\delta^{kl}\right)\nabla^k u^l - \zeta\delta^{ij}(\nabla\cdot u)$$

As for the shear viscosity, one may derive a **new Kubo formula**:

$$\epsilon D_{3/2} = \frac{1}{\mathrm{Tr}\,(-\gamma^0\gamma^0)}\lim_{\omega\to 0}\mathrm{Tr}\left(-\gamma^0\;\mathrm{Im}\int d^dx\, e^{i\omega t}\,\langle S^x_T(x)\bar{S}^x_T(0)\rangle\right)$$

For the aforementioned diffusion constant in the dispersion relation of the phonino, a similar Kubo formula has been given in Kovtun, Yaffe '03. In this notation the respective latter transport coefficient vanishes for (super) conformal theories.

Universal absorption cross sections

For minimally coupled massless scalars the low energy s-wave absorption cross section in a black hole background is universally given by the area of the black hole horizon. A similar universal result was found for minimally coupled massless spin 1/2 fermions (Das, Gibbons, Mathur '96):

$$\sigma_0 = A \quad \leftrightarrow \quad \sigma_{1/2} = 2g(r_H)^{-p/2}A$$

in black hole backgrounds of the form

$$ds^2 = -f(r)dt^2 + g(r)\left(dr^2 + r^2 d\Omega_p^2\right)$$

This is twice the area of the black hole horizon at r_H in a conformally related spacetime! Note also that g(r) does not vanish at the horizon!

Universality proofs

The shear viscosity can be computed from the two-point function of the transverse energy-momentum tensor dual to the transverse graviton:

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int d^dx\, e^{i\omega t}\,\langle[T_{xy}(x), T_{xy}(0)]\rangle$$

The low energy absorption cross section of transverse bulk gravitons is

$$\sigma_{\text{abs},0}(\omega) = -\frac{2\kappa^2}{\omega}\,\mathrm{Im}\,G^R(\omega) = \frac{\kappa^2}{\omega}\int d^dx\, e^{i\omega t}\,\langle[T_{xy}(x), T_{xy}(0)]\rangle$$

Since the transverse graviton obeys the equations of motion of a minimally coupled massless scalar, we get (Kovtun, Son, Starinets '04):

$$\eta = \frac{1}{16\pi G}\sigma_0(0) \;\Rightarrow\; \frac{\eta}{s} = \frac{1}{4\pi}$$

In very much the same way, we may relate the fermionic absorption cross section for vanishing frequency to the Kubo formula in supersymmetric hydrodynamics. The **transverse gravitino** obeys the e.o.m. of a spin 1/2 field (SUGRA gauge invariance). Using this (for vanishing chemical potentials), one gets:

$$\epsilon D_{3/2} = \frac{1}{4\pi G}\sigma_{1/2}(0)$$

Outlook

We have furthermore (in full agreement) computed the supercharge diffusion constants explicitly using the methods of Policastro '08 and Policastro and Kontoudi '12.
Since the transverse gravitino, which is dual to the supersymmetry current, is generically not minimally coupled due to Pauli terms, one may wonder if the universality upholds for finite chemical potential.
Furthermore, one would like to better understand, what the universal quantity in the black hole context means.

Based on: **A universal fermionic analogue of the shear viscosity**; Johanna Erdmenger, Stephan Steinfurt; JHEP 1307 (2013) 018; arXiv:1302.1869