Reflections on F-theory

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I will not be giving references during the talk. There are too many to do so. However, my work is based on many collaborations with Jonathan Heckman and David Morrison and other colleagues and more recently the following papers:

Elliptic Genus of E-strings, arXiv:1412.2324
J. Kim, S. Kim, K. Lee, J. Park, C.V.
Strings of Minimal 6d SCFTs, arXiv:14123152
B. Haghighat, A. Klemm, G. Lockhart, C.V.
Atomic Classification of 6d SCFTs, arXiv: 1502.05405
J. Heckman, D. Morrison, T. Rudelius, C.V.

F-theory is based on the simple idea that branes and geometric singularities are dual to one another. Moreover, the intuition we have for brane dynamics incorporating string dualities is rather limited. Geometry is powerful!

This powerful combination of geometry and brane dynamics explains the many applications that F-theory has found in string theory.

Here I review some of these aspects also pointing out challenging areas where we need more work.

I will focus on two basic applications of F-theory:

1-Particle phenomenology

Realization of GUTs in string theory

2-Strongly coupled quantum systems

Realization of new CFT’s

After that I will move on to challenges:

Motivate a field theoretical version of F-theory where couplings vary over spacetime and transform with duality transformations, and explain what we know about this and why we need to figure out more.

I will state some open problems. Hopefully in the next F-theory meeting someone has solved them!

We have already experimental evidence for some sort of GUT:

1-Approximate unification of coupling constants not too far from the Planck scale.

2-Mass splitting for neutrinos is consistent with Seesaw mechanism, with the new scale in physics not too far from the unification scale.

3-The elegance and simplicity of the matter content and gauge forces.

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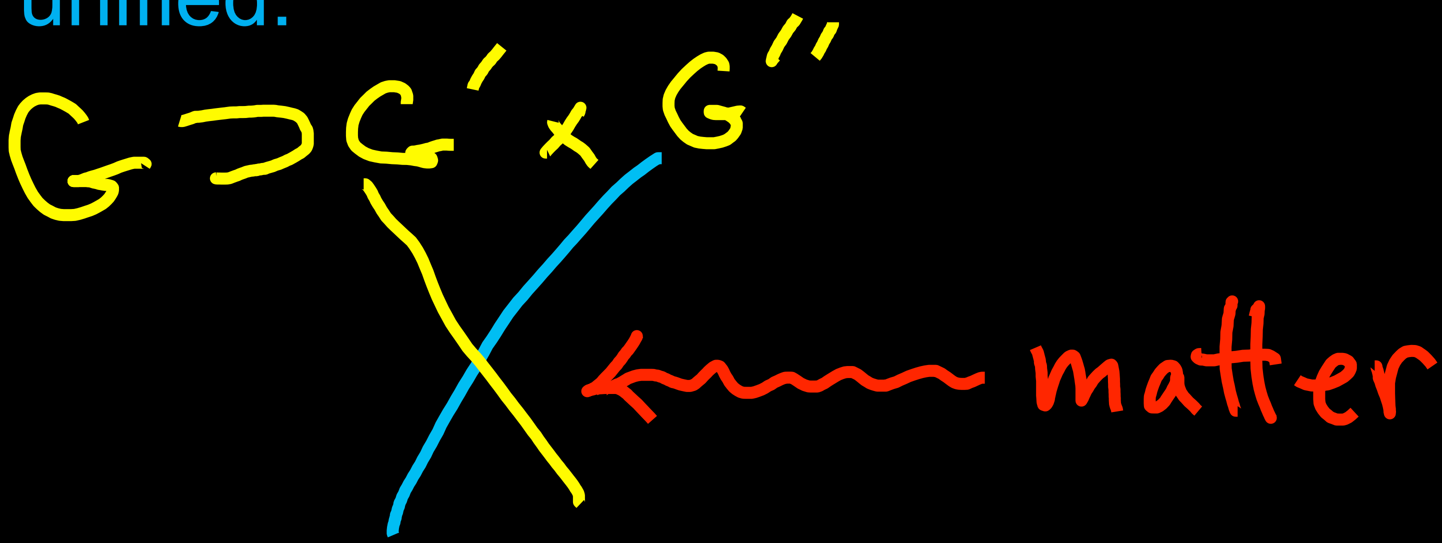

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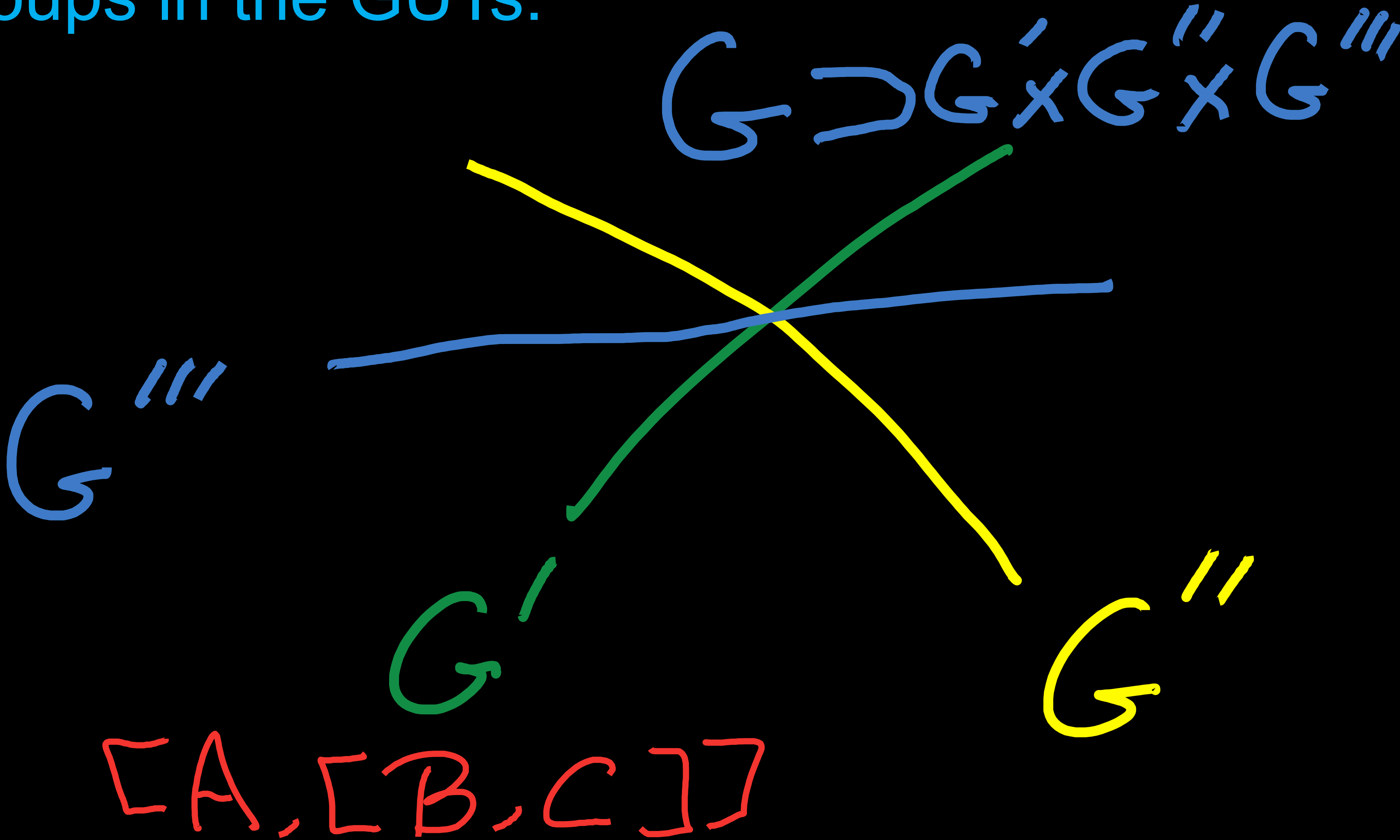


The Yukawa giving the top quark its mass requires epsilon tensor (5-10-10) which cannot be found in classical groups, and has exceptional group as its origin. So we learn that F-theory combined with phenomenology of GUT models predicts we have a point with E-type symmetry.

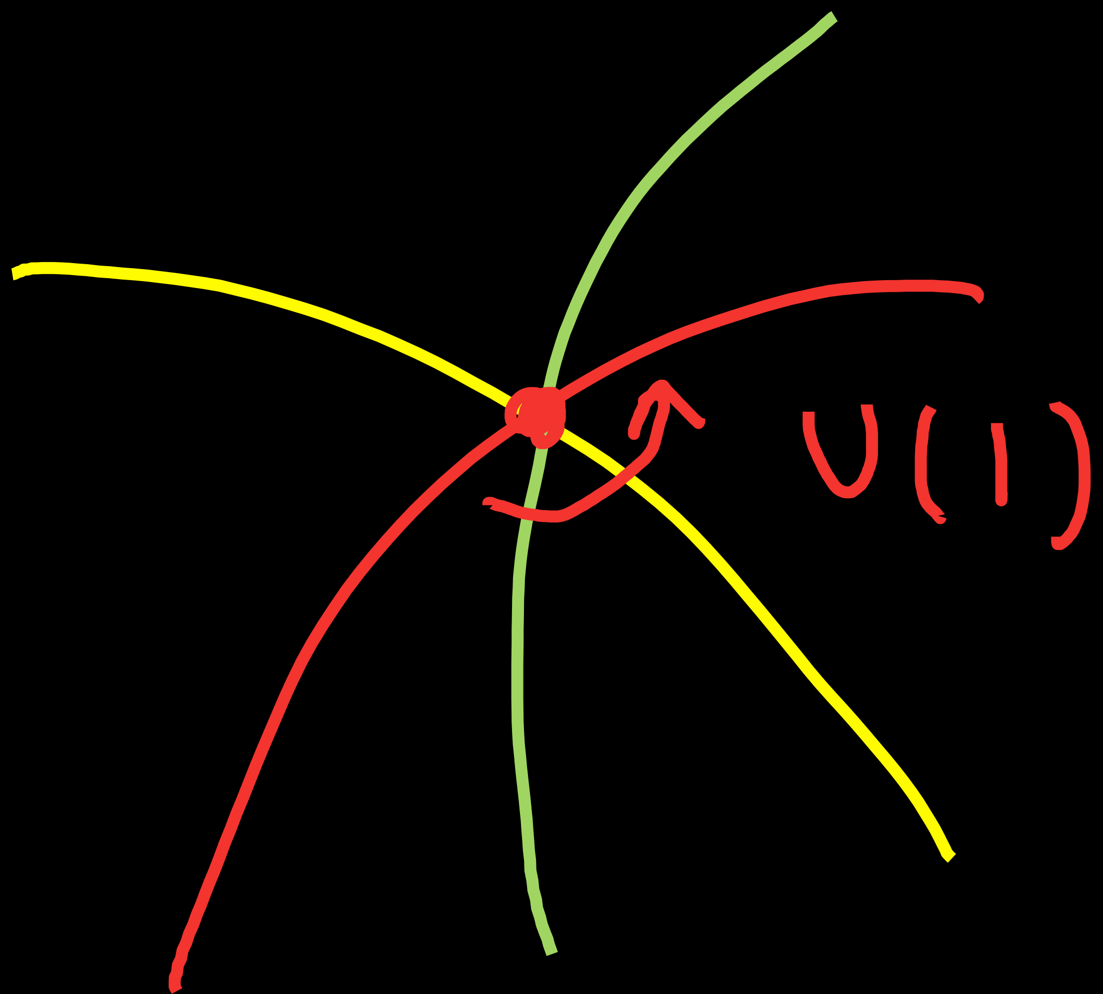
There are interesting implications of this:

The landscape concept typically goes against things being exceptional. Here we seem to have evidence to the contrary.

Flavor Hierarchy: I think this is one area where F-theory and particle phenomenology have a particularly nice fit:

The rank reduction of the mass matrix natural.

The Froggatt-Nielsen mechanism which requires approximate U(1) symmetries get realized as approximate symmetries of rotation group in the extra dimensions.


U(1)

The parameter controlling the U(1) breaking can in particular be related to fluxes in F-theory—leading to non-commutative structure.

What are we to make of LHC results in this context?...too early

If SUSY is found (still some chance) we would be in business!

If no SUSY is found, it would be much harder! But still ideas surrounding hierarchy, require SUSY only near the GUT scale and mass ratios can be extrapolated to our scale even with no SUSY.

Conformal Field Theories

There are two general ways to get interesting CFT's from F-theory:

1-D3 brane probing geometry

2-Geometric Singularities

CFT's from D3 brane probes:

1- 4D Theories with N=2 SUSY:

F-theory on elliptic 3-folds → 8d.
D3 branes probes → N=2 gauge theories.

a) D3 brane probing D4-singularity
SU(2) with 4 flavors

b) D3 brane probing E-type singularity
Minahan-Nemeschansky CFT's with E-global Symmetry

2- 4D SCFT's with N=1 SUSY:

This class can be obtained by D3 branes probing F-theory on elliptic 3- or 4-folds.

This is an interesting class and evidence for existence of non-trivial CFT's have been found. This class is particularly interesting in view of the fact that F-theory with E-point singularity is needed for phenomenology.

This class of theories should be more widely studied both for purely field theoretic reasons and applications.

CFT's from geometric singularities:

The interesting cases would be in 2, 4, or 6d.

The case of 2d and 4d have in general superpotentials which complicates the story. Again, these should be further studied:

We do know that for example in the 4d case with constant elliptic fibration we do get N=2 SCFT's:

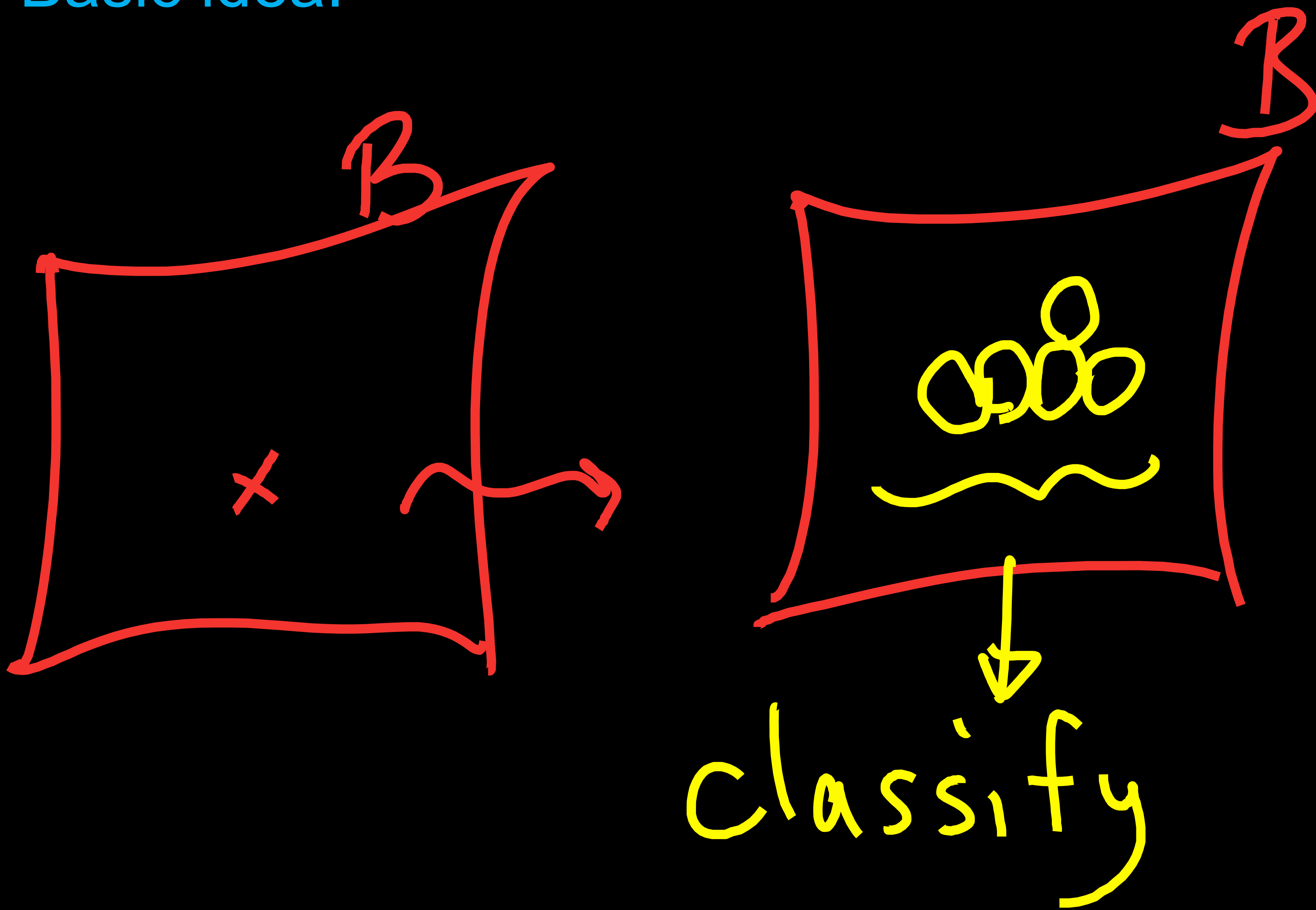
$F(x,y,z,t)=0$ (e.g. $x^n+y^m+z^2+t^2=0$)

6d SCFT from Geometric Singularities

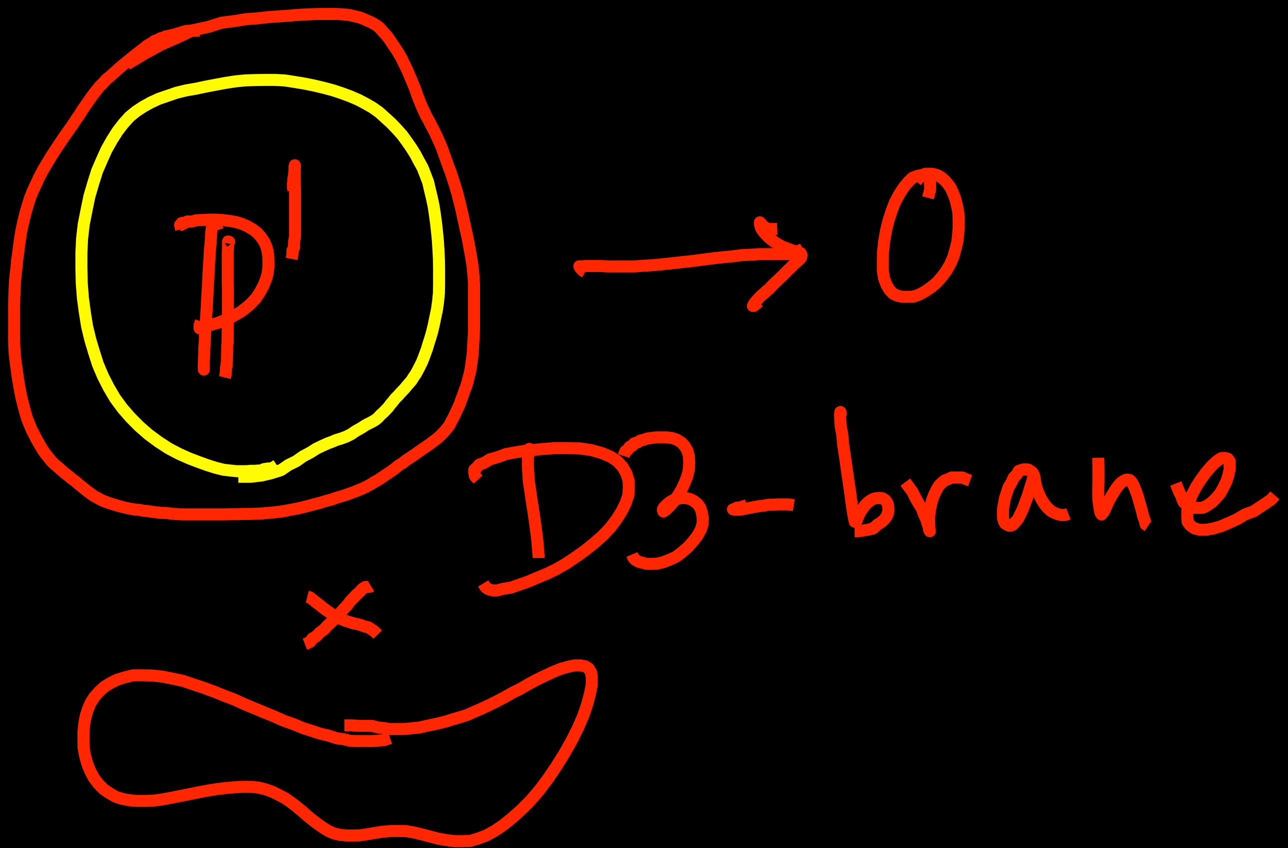
This is a particularly interesting case for F-theory. It corresponds to studying geometric singularities for elliptic CY 3-folds and surprisingly it seems that they are completely classifiable!
(see Jonathan Heckman's talk)

This in particular gives a classification of (1,0) SCFT's in 6d, which is otherwise rather elusive. It is a powerful application of F-theory geometry to unravelling QFT dynamics.

Basic idea:



These theories lead to tensionless strings when the spheres contract:



Novel Field Theoretic Questions

One of the most important novel aspects of F-theory is that the coupling constant of IIB is not constant and jumps by dualities.

It is natural to ask what happens if we do that in ordinary QFT's which depend on some moduli. For example N=2 supersymmetric QFT's often have a non-trivial moduli space. How can we describe the theory which makes the moduli, space dependent?

There are two kinds of things we can consider:

1) How can we compute the partition functions in these cases?

2) If we maintain translation invariance in some subspace, what effective theory do we get? (in particular if we consider R^n x M where M is compact).

These are tough questions. As it turns out for some cases we already know what to do.

Consider: N=4 SYM in 4d and vary its coupling up to SL(2,Z) transformations on the space. We can take the variation to be over a 2d subspace or the full 4d. What can we say in this case?

This is a case which arises in F-theory: The theory on D3 brane. The two situations arise by considering D3 branes wrapping a 2d subspace of the base of F-theory or wrapping a 4d subspace.

Let's consider the partially wrapped D3 brane.
Question: What is the effective 2d theory?

In general the effective theory will have (0,4) supersymmetry, but sometimes it can be enhanced to (0,8) or (4,4):

Examples with $R^2 \times S^2$

1) If S^2 has 24 `cosmic strings' → (0,8)
2) If S^2 has 12 `cosmic strings' → (0,4)
3) If S^2 has 0 `cosmic strings' → (4,4)

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1) If S^2 has 24 `cosmic strings' → O(0)
2) If S^2 has 12 `cosmic strings' → O(-1)
3) If S^2 has 0 `cosmic strings' → O(-2)

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Examples with $R^2 \times S^2$

1) If S^2 has 24 `cosmic strings' → heterotic
2) If S^2 has 12 `cosmic strings' → E-string
3) If S^2 has 0 `cosmic strings' → M-string

In this context the challenge is to deduce what lives on the string.

Heterotic string: D3 brane wrapped on P^1 intersects 24 D7 branes. Naively each intersection point gives a fermion in 1+1 (codimension 8 D-brane intersection).
But we know the 24 D7 branes are not independent, and there are only 20 independent.
20=16+4 complex fermions would work but we need to explain (16L,4 R).

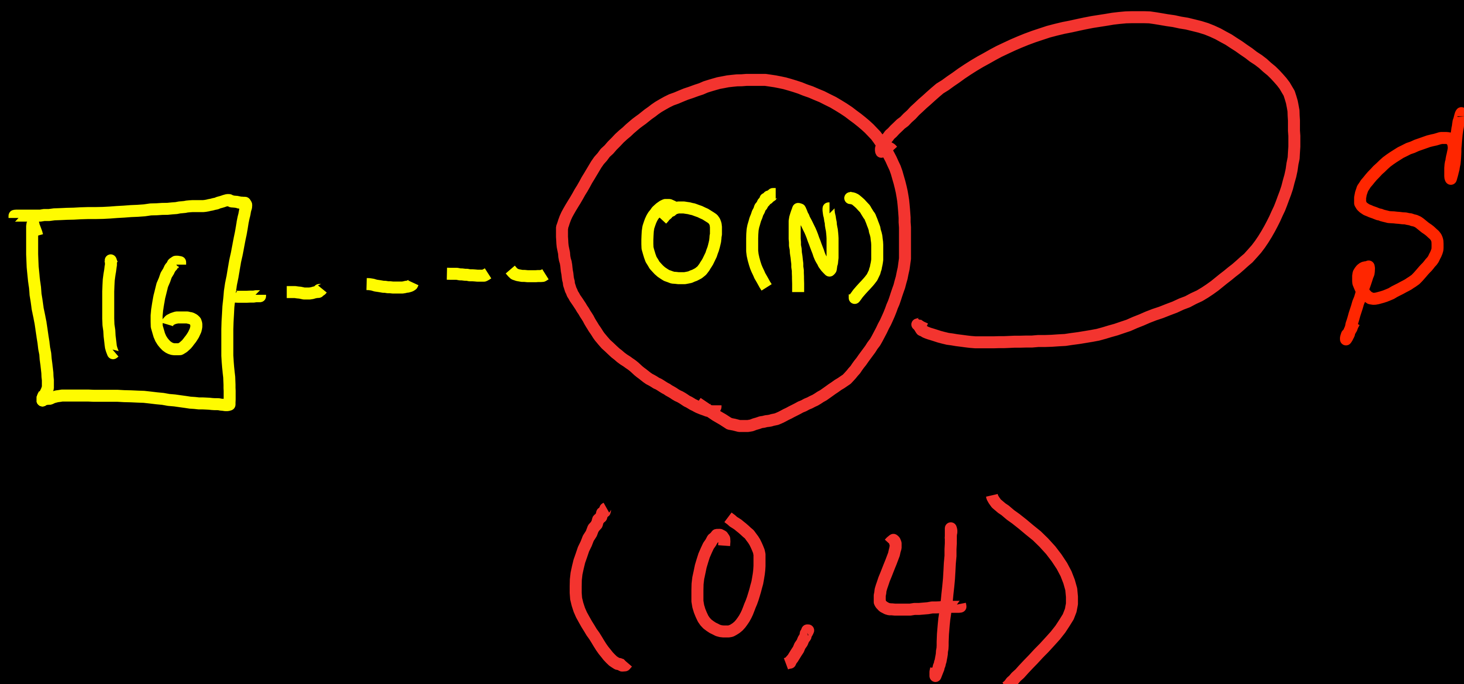
Similarly we need to explain the E-string case.

E-string: 12 D7 branes, but only 10 independent ones, which lead to
10 complex fermions:
10=8L+2R → E8 current algebra +(0,4) SUSY

Even if we explain this, how do we explain the fact that fermion spin structures are summed?

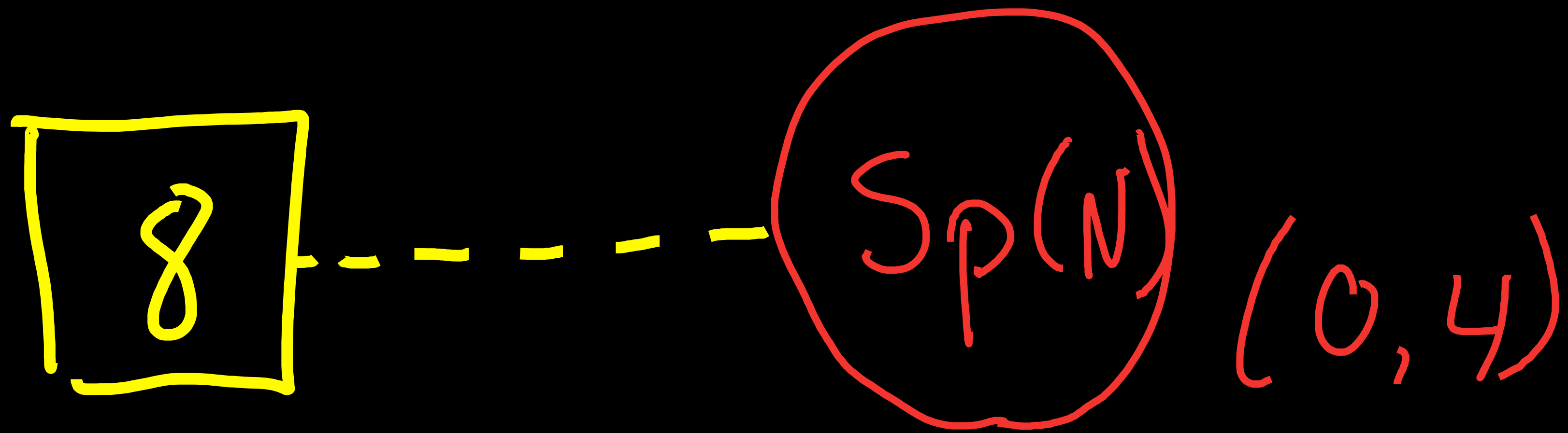
What if we have N D3 branes wrapping P^1, i.e. N E-strings?

One may think this should give a U(N) theory in (1+1) dimension with (8L+2R, N) complex fermions. Not quite. Instead by a chain of dualities we know in this case at least in the IR we get:



For the O(-2) case, i.e. the M-string case, the naïve reduction works and we get a (4,4) U(N) gauge theory. This is the case where elliptic fibration is constant.

How about higher case O(-n)? For n<13 it can arise and gives rise to a CFT. But only for n=4 case, i.e. O(-4) we know the reduction



How about the reduction for other O(-n)?

We do not know!

In this case we can use the duality between F-theory and M-theory to relate the elliptic genus of the effective 1+1 theory to topological A-model on the elliptic 3-fold.
Also related to N=4 topological YM on elliptic fibration over P^1.

Dualities:

F-theory on Y x S^1 → M-theory on Y

D3 brane C x S^1 x R → M2 branes on C x R

topological string

D3 brane C x R^2 → M5 brane on elliptic fibration over C x R^2

D3 brane C x T^2 → M5 branes on elliptic fibration over C x T^2

N=4 YM on elliptic fibration over C

But we do not know how to use this data to decipher the degrees of freedom on the string

Fully wrapped D3 brane:

Arises for superpotentials of 4d N=1 Vacua (Witten)

For D3 brane where tau varies→
Duality with M-theory maps it to M5 brane on elliptic 3-fold with vanishing elliptic class.

For a single D3 brane, this maps to single M5 brane. For many D3 branes we get many M5 branes and this is equally hard to compute.
How do we compute this?

Is there a closed formula for superpotentials generated for F-theory on CY 4-folds?

This should not be too hard for non-compact CY 4-folds.

Challenge: Figure this out!

Any relations to topological strings on 4-folds?

Some Other Theoretical Challenges

The most obvious theoretical question is whether there exists a 12 dimensional theory. Clearly if it exists it should be a 12d theory which does not include 12 dimensional flat space as a solution. Not much has happened in this direction, but I still view this as an interesting question.

Can 10d theory be viewed as a codimension 2 defect in a topological 12d theory?

In the Euclidean version of F-theory, what does compactification on elliptic 6-fold mean? (M-theory duality does not exist!)